**Regular** Article – Theoretical Physics

# Nuclear attenuation of high energy two-hadron system in the string model

N. Akopov, L. Grigoryan, Z. Akopov<sup>a</sup>

Yerevan Physics Institute, Br. Alikhanian 2, 375036 Yerevan, Armenia

Received: 8 June 2006 / Revised version: 21 September 2006 / Published online: 12 January 2007 – © Springer-Verlag / Società Italiana di Fisica 2007

**Abstract.** Nuclear attenuation of the two-hadron system is considered in the string model. The two-scale model and its improved version, with two different choices of constituent formation time and sets of parameters obtained earlier for the single hadron attenuation, are used to describe the available experimental data for the z-dependence of the subleading hadron, whereas satisfactory agreement with the experimental data has been observed. A model prediction for the  $\nu$ -dependence of the nuclear attenuation of the two-hadron system is also presented.

PACS. 13.87.Fh; 13.60.-r; 14.20.-c; 14.40.-n

## 1 Introduction

Nuclear attenuation (NA) of high energy hadrons is a well known tool for the investigation of the early stage of the hadronization process, which cannot be described in the framework of the existing theory of strong interactions (perturbative QCD), because of the major role of "soft" interactions. Nevertheless, there are many phenomenological models, which describe, rather qualitatively, the existing experimental data for single hadron NA [1-13]. Also some predictions for the attenuation of multi-hadron systems leptoproduced in nuclear matter in the framework of the string model [14, 15] were made. It was argued that measurements of NA of multi-hadron systems can remove some ambiguities in a determination of the parameters describing strongly interacting systems at the early stage of particle production: the formation time of hadrons and the cross-section of the intermediate state interaction inside the nucleus. Recently, for the first time, data [16, 17] on the two-hadron system NA ratio measured as a function of the relative energy of the subleading hadron have been published. Therefore, in this work we attempt to describe these data based on the two-scale model (TSM) [5] and the improved two-scale model (ITSM) [13]. We present also predictions for the  $\nu$ dependence of the two-hadron system NA within the same model. Also, possible mutual screening of hadrons in a string and its experimental verification is discussed.

# 2 Theoretical framework

In [14, 15] the process of leptoproduction of the two-hadron system on a nucleus with atomic mass number A was theoretically considered for the first time:

$$l_{\rm i} + A \to l_{\rm f} + h_1 + h_2 + X$$
, (1)

where the hadrons  $h_1$  and  $h_2$  carry fractions  $z_1$  and  $z_2$  of the total available energy. The NA ratio for that process can be expressed in the form

$$R_{\rm M}^{2h} = 2 \, \mathrm{d}\sigma_A \left(\nu, Q^2, z_1, z_2\right) / A \, \mathrm{d}\sigma_D \left(\nu, Q^2, z_1, z_2\right), \qquad (2)$$

where  $d\sigma_A$  and  $d\sigma_D$  are the cross-sections for the reaction (1) on nuclear and deuterium targets, respectively,  $\nu$  and  $Q^2$  denote the energy of the virtual photon and square of its four-momentum,  $z_i = E_i / \nu$ ,  $E_i$  is the energy of the *i*th hadron. One can picture the reaction (1)as shown in Fig. 1. The interaction of the lepton with the nucleon occurs at the point (b, x), where the intermediate state q begins to propagate (b and x are the impact parameter and the longitudinal coordinate of the DIS point). At some points the string breaks, and as a result the first constituents of hadrons  $h_1$  and  $h_2$  are produced at the points  $(b, x_1)$  and  $(b, x_2)$ . Also, at the points  $(b, x_{y1})$  and  $(b, x_{y2})$  a yo-yo of the hadrons  $h_1$ and  $h_2$  is formed (by a "yo-yo" system we mean that a colorless system with valence content and quantum numbers of the final hadron is formed, but without its "sea" partons).

In the string model there are simple connections between the points  $x_{y1} - x_1 = z_1 L$  and  $x_{y2} - x_2 = z_2 L$ , where L is the full hadronization length,  $L = \nu/\kappa$ ,  $\kappa$  is the string tension (string constant). Let us briefly discuss the origin of the connection between the production points of the yo-yo and the first constituent, presented above. In the two-dimensional string models the quark-antiquark pairs arising from the vacuum do not carry energy. The energy

<sup>&</sup>lt;sup>a</sup> e-mail: akopoff@mail.desy.de



**Fig. 1.** Leptoproduction of a two-hadron system on a nuclear target. For details see the text

loss of the leading quark on a unit length is constant and equal to  $\kappa$ . The first constituent of the hadron, which is on the slow end of the open string, absorbs the energy emitted by the leading quark. It receives the necessary energy on the path between its own and the yo-yo production points. Then the vo-vo separates from the main string. This means that for each final hadron there is one arbitrary point – the first constituent production point. In the case of two-hadron system production some models impose certain conditions on the values of  $x_1$  and  $x_2$ . The production of constituents takes place after the DIS, consequently  $x_1, x_2 > x$ ; the next condition is that, according to the ideology of the string model,  $x_1$  and  $x_2$  cannot coincide. In (3)  $x_1 < x_2$  will be used, but this cannot influence the results because (3) is written in a symmetric form with regard to  $h_1$  and  $h_2$ . The NA ratio can be presented in the following form:

$$\begin{aligned} R_{\rm M}^{2h} &\approx \frac{1}{2} \int \,\mathrm{d}^2 b \int_{-\infty}^{\infty} \,\mathrm{d}x \int_x^{\infty} \,\mathrm{d}x_1 \int_{x_1}^{\infty} \,\mathrm{d}x_2 \rho(b,x) \\ &\times \left[ D(z_1, z_2, x_1 - x, x_2 - x) W_0(h_1, h_2; b, x, x_1, x_2) \right. \\ &\left. + D(z_2, z_1, x_1 - x, x_2 - x) W_0(h_2, h_1; b, x, x_1, x_2) \right] \,, \end{aligned}$$

$$(3)$$

where  $D(z_1, z_2, l_1, l_2)$  (with  $l_1 < l_2$ ) is the distribution of the formation lengths  $l_1$  and  $l_2$  of the hadrons (the choice of the function D will be discussed in detail in Sect. 4), and  $\rho(b, x)$  is the nuclear density function normalized to unity.  $W_0$  is the probability that neither the hadrons  $h_1$ ,  $h_2$ , nor the intermediate state leading to their production (initial and open strings) interact inelastically in the nuclear matter:

$$W_0(h_1, h_2; b, x, x_1, x_2) = \{1 - Q_1 - S_1 - [H_1 + Q_2 + S_2 + H_2 - H_1(Q_2 + S_2 + H_2)]\}^{(A-1)},$$
(4)

where  $Q_1$  and  $Q_2$  are the probabilities for the initial string to be absorbed in the nucleus within the intervals  $(x, x_1)$ and  $(x_{y1}, x_2)$ , respectively.  $S_i$  (i = 1, 2) is the probability for the open string containing the first constituent parton for  $h_i$  to be absorbed in the nucleus within the interval  $(x_i, x_{yi})$ , and  $H_i$  (i = 1, 2) is the probability for the  $h_i$  to interact inelastically in the nuclear matter, starting from the point  $x_{yi}$ . The probabilities  $Q_1, Q_2, S_1, S_2, H_1, H_2$  can be calculated using the general formulae:

$$P(x_{\min}, x_{\max}) = \int_{x_{\min}}^{x_{\max}} \sigma_P \rho(b, x) \,\mathrm{d}x\,, \tag{5}$$

where the subscript P denotes the particle (initial string  $Q_i$ , open string  $S_i$ , hadron  $H_i$ , i = 1, 2), the  $\sigma_P$  are the inelastic cross-sections of these objects on a nucleon target  $(\sigma_q \text{ for } Q_i, \sigma_s \text{ for } S_i \text{ and } \sigma_h \text{ for } H_i)$ . All of the notation for  $\sigma$  is taken from [13]. Figure 1 and (4) correspond to the case of TSM. The transition to the case of the improved version – ITSM – is clear. The  $S_i$  are absent,  $\sigma_q$  must be the substitute for  $\sigma^{\text{str}}$ , and the border points between the integration regions of  $Q_i$  and  $H_i$  are  $x_i + c(x_{yi} - x_i)$ , where i = 1, 2; the expressions for  $\sigma^{\text{str}}$  and the parameter c are determined in [13]. The physical meaning of (3) is that the two-hadron system arising as a result of DIS in the nucleus is not absorbed in the nuclear matter.

## **3** Experimental situation

Recently the HERMES Collaboration has obtained, for the first time, data on double hadron attenuation [16, 17]. The following double ratio for leading and subleading hadrons has been measured:

$$R_{\rm M}^{2h}(z_2) = \frac{(\mathrm{d}^2 N(z_1, z_2)/\mathrm{d} N(z_1))_A}{(\mathrm{d}^2 N(z_1, z_2)/\mathrm{d} N(z_1))_D}, \qquad (6)$$

where  $z_i = E_i/\nu$ ,  $E_i$  is the energy of *i*th hadron, *A* and *D* denote that the interaction takes place on nuclear and deuterium targets,  $d^2N(z_1, z_2)$  is the number of events with at least two hadrons (leading and subleading hadrons in one event). According to the experimental conditions the leading hadron must have  $z_1 > 0.5$ , and  $dN(z_1)$  is the number of events with at least one hadron with  $z_1 > 0.5$ . In this experiment the double ratio  $R_{\rm M}^{2h}(z_2)$  was considered as a function of  $z_{\rm subleading} = z_2$ . Two sets of experimental data are presented:

- 1. the leading–subleading combinations ++, --, +0, 0+, -0, 0-, 00 only;
- 2. all hadron pairs except those with invariant mass near  $\rho^0$ .

In our work, we use the first set, because, in our opinion, it contains less contributions from hadrons produced in diffractive and diffractive dissociation processes. The relevant region for the investigation of the two-hadron system NA ratio as a function of  $z_2$  is the one between  $z_2 = 0.1$ and  $z_2 = 0.4$ . At  $z_2 < 0.1$  the contributions from the slow hadrons coming from the target fragmentation become large. At  $z_2 > 0.4$  it is difficult to distinguish a leading and subleading hadron because in this region  $z_1 \approx z_2 \approx 0.5$ .

Let us also consider the following question – to what extent are these data free from contributions of diffractive  $\rho^0$ -mesons? For events  $dN(z_1)$  corresponding corrections were made by experimentalists. Here we discuss the situation with  $d^2N(z_1, z_2)$  only. At first glance the diffractive  $\rho^0$ -mesons do not give contributions in these events because of the special conditions at the choice of pairs of hadrons. They ensure that both hadrons in a pair cannot be produced from decay of one  $\rho^0$ -meson, but do not forbid that one of the hadrons in the event was produced as a result of breaking of a diffractive  $\rho^0$ -meson. Then, another hadron could be produced after the  $\rho^0$ -meson final state interaction.

#### 4 Results and discussion

We have performed calculations for  $R_{\rm M}^{2h}(z_2)$  in the frame-work of TSM and its improved version. The ratio  $R_{\rm M}^{2h}$  is a function of the three variables  $z_1, z_2$  and  $\nu$ , but for comparison with the available experimental data we present  $R_{\rm M}^{2h}$  as a function of  $z_2$  only, after integration over the other variables according to the experimental conditions  $(0.5 < z_1 < 1 - z_2, 7 < \nu < 23.5 \text{ GeV}, p_h > 1.4 \text{ GeV}/c)$ . In this paper we also present model predictions for  $R_{\rm M}^{2h}$  as a function of  $\nu$ , while performing integrations over  $z_1, z_2$ in the corresponding kinematic regions. They are chosen to be close to the conditions of the available experimental data  $(0.5 < z_1 < 0.9, 0.1 < z_2 < 1 - z_1)$ . The upper limit of  $z_1 < 0.9$  relates to the case of two hadrons in final state, as well as for the single hadron this limit has to be equal to unity. All theoretical curves presented here were obtained with the assumption that the final hadrons are pions. Two expressions for the constituent formation time (CFT) for the two-hadron system were used. As a first expression for CFT the distribution function  $D(z_1, z_2, l_1, l_2)$ for the formation lengths  $l_1$  and  $l_2$  calculated in the framework of the Lund model was used. It corresponds to (4.1)from [15] and will be denoted CFT1. The second expression was taken in a form following [18]. It is supposed that the leading hadron which contains the leading quark has been produced at the fast end of the string. For the case of the two-hadron system production we extend this consideration, in the sense that the leading and subleading hadron systems are produced at the fast end of the string. Then as the CFT for the first produced constituent of the first hadron we have  $\tau_{c1} = (1 - z_1 - z_2)L$ , and for the first produced constituent of the second hadron  $\tau_{c2} = (1 - z_2)L$ (denoted CFT2). This means that  $h_1$  and  $h_2$  are the last but one and the last hadron from the slow end of the string, and consequently the second and first hadron from the fast end of the string. The distribution function  $D(z_1, z_2, l_1, l_2)$ 

in this case turns into two  $\delta$  functions. We would like to remind the reader that in relativistic units  $\tau_{ci} = l_i$ , i = 1, 2. In this work, for the calculations we use the parameters obtained in [13] for the case of single hadron attenuation, by applying our model to the corresponding HERMES data. We consider the results of the fit reasonable enough, because the values of the cross-sections and the parameter c obtained in the fit turned out to be close to expectations. After the DIS, a color string is stretched between the knocked out quark and the nucleon remnant, a string which has a length on the order of the hadronic size (because of confinement), and the transverse size is on the order of  $1/\sqrt{Q^2}$ , which is essentially smaller than the hadronic size. This means that  $\sigma_q$  must be much smaller than typical hadronic cross-sections. From Tables 1–3 of [13] we see that this is true for all versions of the model. In the TSM it was also expected that, after the first constituent production, the resulting open string will interact with a cross-section close to the hadronic one. From Table 1 we see that it is also true independent of the choice of the constituent formation time. In ITSM it is expected that, soon after production of the first constituent, the cross-section of the open string will reach the value of a hadronic cross-section, i.e. the parameter c must be small compared to unity. This expectation is also realized in all versions with reasonable values of  $\chi^2$ .

Three sets of parameters (including the nuclear density functions) corresponding to the minimum values of  $\chi^2$  from Tables 1–3 [13] were used for the calculations. The value of the string tension was fixed at  $\kappa = 1 \text{ GeV/fm}$ . Results



**Fig. 2.** Double ratio  $R_{\rm M}^{2\rm h}$  as a function of  $z_2$ . The explanation for the points and curves is in the text

for the double ratio  $R_{\rm M}^{2h}$  as a function of  $z_2$  are presented on Fig. 2. On panels a, c and e three options of the theoretical curves are shown: solid curves correspond to the TSM with CFT1; dashed curves correspond to the ITSM with CFT2; dotted curves correspond to the ITSM with CFT1. According to the ideology of the string model, the transverse size of the string is much less than the longitudinal one. This means that the hadrons produced from the string have close impact parameters and could partly screen each other, which in turn must lead to the weakness of NA (partial attenuation). To study this effect and to compare with the basic supposition that two hadrons attenuate independently (full attenuation), we consider partial attenuation in its extreme case, when two hadrons fully screen each other, and as a result the two-hadron system attenuates as a single hadron. The results of the calculations within these conditions are shown in panels b, d and f. The two-hadron system will attenuate as a single hadron also when the two final hadrons appear as a result of breaking of one of the resonances. For instance, the combinations +0 (-0) and 0+ (0-) can be obtained as products of the decay of the  $\rho^+$ - ( $\rho^-$ -) mesons produced via a fragmentation mechanism in the nucleus, when the decay occurred outside of the nucleus. Comparison with the experimental data for the  $z_2$ -dependence shows that the difference between the versions is smaller than the experimental errors; consequently, different versions of the model cannot be distinguished by means of a comparison with these data. Calculations with full and partial NA give close results. The theoretical curves quite satisfactorily describe the data for nitrogen. In the case of krypton and xenon targets the situation is more ambiguous. While the three middle points are described satisfactorily, the two extreme points corresponding to lower and higher values of  $z_2$  are in much worse agreement. In our opinion, a possible reason for that could be that our model does not contain the necessary ingredients for a quantitative description of these points.

Let us briefly discuss the mechanisms which, we believe, give a considerable contribution in the extreme points discussed above but are not included in present model. A first mechanism which can lead to the increasing of the NA ratio at lower value of  $z_2$  is that part of the subleading hadrons in nucleus are protons, which are produced in abundance at small z, and in this region they have a value of the NA ratio larger than unity. We will try to estimate the contribution of protons in Appendix A. A second mechanism which can lead to the increasing of the NA ratio at lower  $z_2$  is the rescattering of produced hadrons in the nucleus; as a result hadrons spend part of their energy for the production of slow hadrons. Consequently, more slow hadrons arise in a nucleus than in deuterium, and, despite absorption, the multiplicity ratio in this region can become close to and even larger than unity. Our model does not take into account the final state interactions of the produced hadrons; consequently, at present we cannot calculate or estimate the contribution of this mechanism. Concerning the second extreme point at higher values of  $z_2$ , which is equal  $z_2 = 0.44$ , we suppose that the double hadron attenuation ratio in this point is on the order of unity, because there are two more additional mechanisms, which are not



**Fig. 3.** Double ratio  $R_{\rm M}^{2\rm h}$  as a function of  $\nu$ . The explanation for the curves is in the text

included in the present model. The first one had to do with the pairs of pions appearing as a result of breaking of coherently produced diffractive  $\omega$ -mesons, for which the coherent cross-section depends proportionally on the atomic mass number by  $A^2$ . As a result, the NA ratio for heavy nuclei increases. It is very difficult to estimate the contribution of this mechanism without the implementation of additional free parameters. The second mechanism is connected with the smallness of the integration region over  $z_1$ , which in the case of  $z_2 = 0.44$  is equal to  $0.06^1$ . The NA ratio for the two-hadron system is proportional to the ratio of integrals over  $z_1$  on nucleus and deuterium. Taking into account Fermi motion or nucleon-nucleon correlations can lead to an extension of the integration region in the nucleus and as a result to the increasing of the NA ratio (see the details in Appendix B). The model gives close results for the two-hadron system NA ratio for the krypton and xenon targets.

Figure 3 shows the prediction of the model for the  $\nu$ dependence of the double ratio  $R_{\rm M}^{2h}$  for nitrogen, krypton and xenon targets. On panels a, c and e three varieties of the theoretical curves are shown: solid curves correspond to the TSM with CFT1; dashed curves correspond to the ITSM with CFT2; dotted curves correspond to the ITSM with CFT1. On panels b, d and f are shown the same curves calculated with the additional condition that only the first produced hadron attenuates (partial atten-

<sup>&</sup>lt;sup>1</sup> This value of 0.06 is in fact related to the case of only two hadrons in the final state; more hadrons in the final state lead to a decrease of the integration range.



Fig. 4. Double ratio  $R_{\rm M}^{\rm 2h}$  as a function of  $\nu$ . The explanation for the curves is in the text. *Lower curves* correspond to the case that two hadrons attenuate independently (full attenuation) and *upper curves* correspond to the case that only the first produced hadron attenuates (partial attenuation)

uation). It is easy to see that curves corresponding to the full and partial attenuation have a different behavior at low values of  $\nu$ . In Fig. 4 is shown, as an example, the case of krypton only. The meaning of the curves is as in Fig. 3. Lower curves correspond to the case of full attenuation and upper curves correspond to the case that only the first produced hadron attenuates (partial attenuation). The measurement of the NA ratio in the region of  $\nu$  from 3 GeV to 10 GeV is useful to verify the supposition of possible mutual screening of hadrons in a string. We think that such an experiment can be useful for a comparison with the results obtained at RHIC by the STAR Collaboration [19], which state two hadrons from one jet to be absorbed more weakly than two hadrons from away-side jets.

# **5** Conclusions

- The string model [14, 15] gives a natural and simple mechanism for the description of the two-hadron system NA, which allows one to describe the available experimental data for the z-dependence of the subleading hadron on a satisfactory level, using the sets of parameters obtained in [13] for single hadron NA.
- Comparison with the experimental data for the  $z_2$ -dependence shows that the difference between versions of

the model is smaller than the experimental errors; consequently, they cannot be distinguished by means of a comparison with these data.

- The double ratio considered as a function of the partial energy of the subleading hadron  $z_2$  has a weak sensitivity to the mutual screening of hadrons.
- It is of certain interest to also study other aspects of the two-hadron system production in a nuclear medium. In particular, we propose to measure the  $\nu$ -dependence of NA, because, as we have shown, it is more sensitive to the mutual screening of hadrons than the  $z_2$ -dependence. Investigation of the  $\nu$ -dependence in the region of  $\nu$  from 3 GeV to 10 GeV will allow for better understanding of questions connected with the possible mutual screening of hadrons in the string. The corresponding measurements can be performed at HERMES and JLab.
- Estimations show that agreement with the experimental data can be improved by means of the inclusion of additional mechanisms which were not included in the model we have presented.

A last remark concerning the description of the data for the two-hadron system attenuation in other models: there are at least two theoretical works which attempt to describe the data for thetwo-hadron system NA; the first of them based on the BUU transport model [20], and the second one based on the so called energy loss model [21, 22].

# Appendix A

In this appendix we will discuss the considerable difference between our model and the experimental data at the point  $z_2 = 0.09$  for a krypton nucleus, and we will try to understand the cause of this discrepancy. Our model, as mentioned above, takes into account pions only. But in heavy nuclei many slow protons are produced, in addition to pions (in this discussion we do not distinguish kaons from pions). We want to show that, by including in the consideration these protons, one can improve the agreement with the data. Let us present the two-hadron system NA ratio in the form

$$R_{\rm M}^{2h} = (1 - \alpha) R_{\rm M}^{2\pi} + \alpha R_{\rm M}^{\pi P} , \qquad (A.1)$$

where  $\alpha$  is the part of the events which contains pairs consisting of a fast pion and a slow proton,  $R_{\rm M}^{2\pi}$  and  $R_{\rm M}^{\pi P}$  are two-hadron system NA ratios in the case when a pair consists of two pions and pion and proton, respectively. It is convenient to introduce a parameter  $\beta = R_{\rm M}^{\pi P}/R_{\rm M}^{2\pi}$ . Then  $R_{\rm M}^{2h}$  can be rewritten in the form

$$R_{\rm M}^{2h} = (1 + \alpha(\beta - 1))R_{\rm M}^{2\pi}.$$
 (A.2)

The parameter  $\beta$  can be defined from the data on single hadron attenuation, if one can assume that the fast and the slow hadrons in the event are produced independently (the correlation between them can be neglected). Then  $\beta \approx R_{\rm M}^P/R_{\rm M}^{\pi}$ , where  $R_{\rm M}^P$  and  $R_{\rm M}^{\pi}$  are the single hadron NA ratios for proton and pion respectively. In addition to the  $z_2 = 0.09$  point we make estimates for the next point,  $z_2 = 0.15$ , also, because contributions of protons in this point can still be considerable. Using the available data for krypton [23], we have extrapolated to the point z =0.09 (0.15) and obtained  $R_{\rm M}^P \approx 1.5$  (1.425) and  $R_{\rm M}^\pi \approx 0.85$ (0.85), which gave  $\beta \approx 1.765$  (1.676). Then we determined the parameter  $\alpha$  from (A.2). For  $z_2 = 0.09$  (0.15) the model gives  $R_{\rm M}^{2\pi} \approx 0.89$  (0.87), while the experimental value is  $R_{\rm M}^{2h} \approx 1.2$  (0.92), which gives  $\alpha \approx 0.47$  (0.087). The results of the calculations with the model improved by means of inclusion of the protons, as was discussed above, are shown in Fig. 5a for the krypton case. Three theoretical curves are shown: the solid curve corresponds to the TSM with CFT1; the dashed curve corresponds to the ITSM with CFT2; the dotted curve corresponds to the ITSM with CFT1. Such a value of  $\alpha$  for the first point seems to be too big. But one can note that still this correction works in the right direction and improves the agreement of the theory with the experimental data even if the actual value of the parameter  $\alpha$  is smaller than the one following from our estimate. In Fig. 5b the dependence on the value of the parameter  $\alpha$  at  $z_2 = 0.09$  is shown. As an example we take the case of TSM with CFT1 calculated for three values of  $\alpha$  equal to 0.47 (upper curve), 0.30 (middle curve), 0.10 (lower curve). Simultaneously we proportionally changed the values of  $\alpha$  in the next experimental point also. The estimates with other versions of the model give close results.



**Fig. 5.** Double ratio  $R_{\rm M}^{\rm 2h}$  as a function of  $z_2$ . The result of the proton inclusion in the model is demonstrated (see for details Appendix A). The explanation for the points and curves is in the text.

### Appendix B

As mentioned in the text, the Fermi motion of nucleons in the nucleus and the presence of nucleon–nucleon correlations (fluctons) can improve the agreement with the experimental data at  $z_2 = 0.44$  by means of extending the range of integration.

Let us first consider the influence of the Fermi motion on the increase of  $z_{\text{max}}$ . We would like to remind the reader that in the case of scattering on a nucleon in rest  $z_{\text{max}} = 1$ . The total center of mass energy of the secondary hadrons is W, and in the case of taking Fermi momentum into account we denote it  $W_{\text{F}}$ . Then, after averaging over the angle between virtual photon and nucleon momenta, it can be presented as

$$W_{\rm F}^2 = M^2 - Q^2 + 2E_{\rm N}\nu = M^2 - Q^2 + 2M\nu_{\rm eff}\,,\qquad({\rm B.1})$$

where M and  $E_N$  are mass and energy of the nucleon respectively,  $\nu_{\text{eff}}$  is the value of  $\nu$  which gives to the nucleon in rest the value of W equal to  $W_{\text{F}}$ . Then for  $z_{\text{max}}$  we obtain

$$z_{\rm max} = \frac{\nu_{\rm eff}}{\nu} = \frac{E_N}{M} \,. \tag{B.2}$$

Now we can estimate the influence of the Fermi momentum. If we take the average Fermi momentum for a medium and heavy nucleus to be equal to 0.25 GeV/c [24], then  $z_{\text{max}} = 1.035$  and the NA ratio, taking into account that also the integration region for single hadrons will be increased (about 10%),  $R_{\text{M}}^{2h}$  at  $z_2 = 0.44$ , must be multiplied by a factor of 1.42.

As an alternative mechanism the nucleon–nucleon correlations could be considered in the framework of the flucton mechanism. That means that a virtual photon scatters on fluctons with the masses M, 2M, 3M, etc. If one takes into account only one- and two-nucleon correlations, one obtains

$$z_{\rm max} = \frac{\nu_{\rm eff}}{\nu} \approx 1 + \alpha_{\rm fl} \,, \tag{B.3}$$

where  $\alpha_{\rm fl}$  is the probability of scattering on the two-nucleon flucton. If, for the sake of an estimate, one takes  $\alpha_{\rm fl} = 0.01$ then  $z_{\rm max} = 1.01$ , and the NA ratio  $R_{\rm M}^{2h}$  at  $z_2 = 0.44$  must be multiplied by a factor of 1.17. We see that both mechanisms give considerable improvement for the last  $z_2$  point.

Acknowledgements. We would like to acknowledge P. Di Nezza, who has initiated the double hadron measurement at HERMES, as well as many other colleagues from the HERMES Collaboration for fruitful discussions.

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